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BUBBLES WITH AN O(3) SYMMETRIC SCALAR FIELD IN CURVED SPACETIME[1]

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Abstract

We study the first-order phase transition in a model of scalar field with $O(3)$ symmetry coupled to gravity, and, in high temperature limit, discuss the existence of new bubble solution with a global monopole at the center of the bubble.

Since the first-order phase transition was formulated in the context of Euclidean path integral [1], it has attracted much attention as a possible resolution of cosmological problems. It is widely believed that the first-order phase transition is described by the formation and growth of bubbles in which no matter lumps remain.

In this note, we shall discuss a possibility of a new bubble solution that the global monopole is supported at the center of it [2, 3]. Such a new type of solution was first discovered in a flat spacetime by one of the present authors [2]. The $O(3)$-symmetric scalar field in the presence of gravity at high temperature is described by the action,

$$S_E = \int_0^{\frac{1}{T}} dt \int d^3x\sqrt{g}\left\{-\frac{R}{16\pi G} + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^a + V(\phi)\right\}.$$

In our calculation we assume a sixth-order scalar potential such as

$$V(\phi) = \lambda\Big(\frac{\phi^2}{v^2} + \alpha\Big)(\phi^2 - v^2)^2, \qquad (0 < \alpha < 0.5)$$

however, our argument does scarcely depend on the detailed form of scalar potential if it has a true vacuum at ϕ_- with $V(\phi_-) = 0$ and a false vacuum at ϕ_+ with $V(\phi_+) > 0$. Since time-dependence can be neglected in high-temperature limit, the spherically symmetric bubble solutions are constituted under the hedgehog ansatz, $\phi^a = \phi(r)(\sin n\theta\cos n\varphi,\ \sin n\theta\sin n\varphi,\ \cos n\theta)$, where n has to be 0 or 1 for the sake of the spherical symmetry. We present a numerical solution of scalar field in Fig.1 and distribution of the energy density in Fig.2 for

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$\lambda = 1,\ \alpha = 0.1,\ v = 0.1M_{Pl}$. Fig.1 shows that the $n = 0$ bubble is nothing but the normal high-temperature bubble in a real singlet scalar model [4] where its central region is in true vacuum, but the center of the $n = 1$ bubble should be in false vacuum because of the winding between $O(3)$ internal symmetry and spatial rotation. From the profiles of energy density ρ in Fig.2, we find that a global monopole is formed at the center of the $n = 1$ bubble. This global monopole is surrounded by the inner wall ($R/H^{-1} \sim 0.1$) which distinguishes the lump from the true vacuum region ($0.2 < R/H^{-1} < 0.5$). The long-range tail of global monopole, $\rho \sim 1/r^2$, renders the spacetime between the inner and outer walls flat with the deficit (solid) angle $\Delta = 8\pi^2 G v^2$. When we consider the object in Planck scale, the spacetime of monopole will be closed by itself. However, if we consider a non-Abelian gauge field as well, just as in the standard model or grand unified theories, we will find a formation of black hole carrying a magnetic charge [5].

We now turn to the evaluation of nucleation rate, $\Gamma \sim Ae^{-B}$. Since the $n = 1$ bubble consumes additional energy to support a global monopole, its size is larger than that of $n = 0$ bubble (see Fig.2), and then B_1 is always larger than B_0 regardless of the strength of gravity and of the shape of the scalar potential, where B_n is the value of Euclidean action of n bubble. As an example, we show a numerical result of $B' \equiv (T/v)B$ in Fig.3 ($\lambda = 1,\ \alpha = 0.1$) and Fig.4 ($\lambda = 1,\ v = 0.1M_{pl}$). The exponential part of nucleation rate, $\Gamma^{(n)} \sim e^{-B_n}$, tells us that the $n = 0$ bubble is always more favorable than the $n = 1$ bubble. However, we find that gravity enhances the decay rate, which is consistent with the results for a singlet scalar field at zero temperature [6], and for large α (small potential barrier), the decay rate of the $n = 1$ bubble becomes comparable to that of the $n = 0$ bubble, i.e., the relative decay rate $\Gamma^{(1)}/\Gamma^{(0)}$ may be determined by the ratio of prefactors A_n (see Ref.2). We may need further analysis including high temperature quantum correction effect on the potential to give a definite answer.

Though the high-temperature bubbles are given by static solutions when they are nucleated, they will immediately start to grow by the following reasons; one is some combustion process when the environment keeps the temperature high [7, 4] and the other is the recovery of zero-temperature classical dynamics due to the expansion of background universe dominated by radiation. We only take into account the latter effect here. The motion of outer wall of $n = 1$ bubble resembles that of $n = 0$ bubble. For the matter droplet inside $n = 1$ bubble, the global monopole is stable under the influence of gravitation when the phase transition scale is lower than Planck scale, as shown in Figs.5 and 6 ($\lambda = 1,\ \alpha = 0.2,\ v = 0.1$). In Fig.6 we present trajectories of positions of $\phi = 0.5v$. However, if we consider this object in Planck scale, an issue that whether the core site shows a topological inflation [8] or it shrinks to a black hole or a wormhole [9] needs further study.

References

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Fig.1

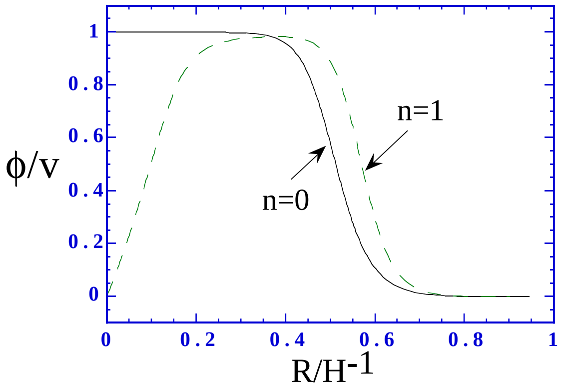

Fig.2

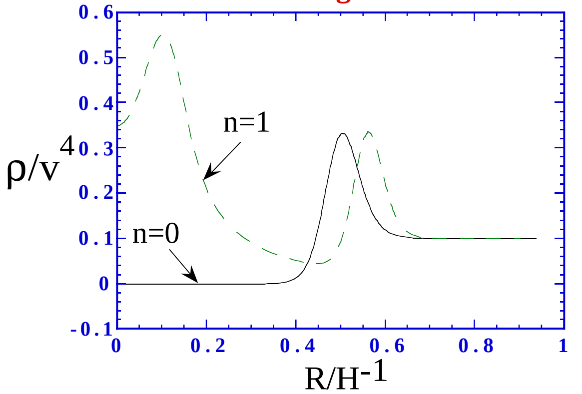

Fig.3

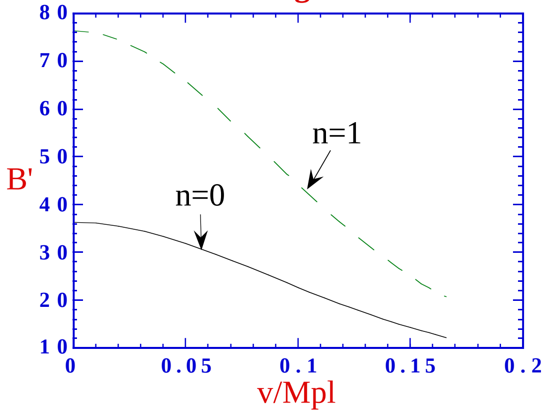

Fig.4

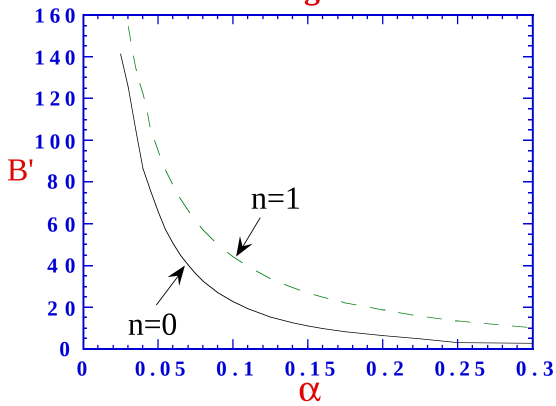

Fig.5

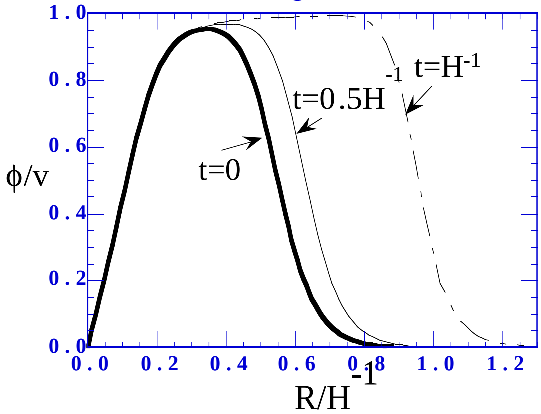

Fig.6